

Two-Dimensional Geostatistical Methods — Theory, Practice and a Case Study from the 1A Shoot Nickel Deposit, Leinster, Western Australia

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ABSTRACT

Two-dimensional (2D) estimation approaches have significant advantages over three-dimensional (3D) approaches when dealing with a narrow vein- or layer-like geometry. The main motivation for 2D estimation is that bias due to non-additivity of grade is avoided. The various aspects of 2D estimation are illustrated by use of a case study from the 1A Shoot Nickel deposit at Leinster, Western Australia. The width of the 1A mineralisation is variable, and so are the angles of drilling intersection. Consequently, the variable of interest (in this case nickel grade) is not suitable for direct kriging because it is defined on non-uniform support. However, grade can also be defined as the ratio of two variables that are suitable for direct kriging: thickness and accumulation. In the 2D approach, tonnage equates to thickness and metal equates to the product of grade and thickness, also called 'accumulation'. In the simplest approach we can independently kriging accumulation and thickness and derive the grade by the ratio of the two resultant estimates. The thickness and accumulation variables must be in a consistent projection orientation, for example a horizontal projection in the case of steeply inclined mineralisation like 1A. Because the density is not constant across the 1A (and most other) deposits, the issue of density weighting is also discussed. The impact of the correlation between grade and length, or grade and density are important issues in implementation of 2D estimation. The circumstances under which these correlations need to be taken into account (by co-kriging) are reviewed. Finally some comments are made on the final use of the resultant 2D estimate for mine planning purposes.

INTRODUCTION

This paper sets out the main considerations and technical details for estimation of resources in two-dimensions (2D) by geostatistical methods. Reference is made throughout to a case study from the 1A Shoot nickel deposit at Leinster, Western Australia.

When undertaking resource estimation on any deposit with narrow vein- or layer-like geometry, the variable of interest (say grade) is not a suitable variable for direct kriging. This is because grade of mineralised intercepts is clearly defined on varying supports. But grade can also be defined as the ratio of two other variables (thickness and accumulation – the product of grade by thickness potentially weighted by bulk density), which are amenable to direct kriging (Chiles and Delfiner, 1999). There is also an operational issue to consider, because the variables of economic interest, ie those upon which economic decisions and optimisations will be made, are actually the projected horizontal thickness (tonnage) and the accumulation (metal content) and *not* the grade. In these situations grade is usually of secondary interest.

The complexity of nickel distribution within the case study deposit (1A Shoot) is such that no internal across strike selectivity is likely to be possible. The estimation of the geometry and overall (nickel) metal content is critical for the economic evaluation of the project, and a 2D approach is therefore suitable.

The appropriateness of a 2D approach, in the interests of avoiding biases due to violation of additivity, is beyond debate. Additivity refers to the following property of a variable: that the linear average of its values will result in a variable with similar physical meaning (see Journel and Huijbregts, 1978 pp 199-200, for discussion of additivity).

The definition of the new variables (thickness and accumulation) is straightforward: it simply requires a careful consideration of not only the grade and length of the mineralised intercepts but also their orientation, if a meaningful, appropriate projection is to be decided upon. Once these variables are defined, the tools of linear geostatistics can be deployed at will.

It is worth stating that additional practical problems with 3D approaches when applied to narrow seams, layers or veins arise from the difficulties of dealing with geometry. Specifically:

1. to capture the essential geometry of most veins or layers, the block size must be too small (from an estimation variance point of view);
2. the alternative of estimating large blocks and using sub-blocking (assigning parent cell grades) or, equivalently, 'block partials' (ie percentages) is also problematic because the kriging assumes the support of the parent blocks (and in these cases this is generally too large, leading to excessive smoothing); and
3. the appropriate composite length to employ for 3D estimation is also problematic: there is no way of correctly and exactly honouring the hangingwall-to-footwall sample dimensions.

These problems along with additivity issues are all overcome when a 2D approach is adopted.

AN ILLUSTRATIVE EXAMPLE OF RESOURCE MODELLING IN 2D

When dealing with a 2D layer, grade is not a suitable input for direct kriging because it is defined on varying 'support'. However, the grade can also be defined as the ratio of two variables that are clearly both amenable to direct kriging, namely the thickness and accumulation (grade multiplied by thickness).

The problem of 'additivity' is of critical importance and motivates the 2D approach. On variable support, the grade of unequal sample lengths cannot be linearly averaged to obtain a correct result because the outcome of the linear averaging is not a grade.

The important concepts are best appreciated with the aid of an illustrative example. Figure 1 is a diagrammatic plan view of a 2D layer with known lateral extent. The objective of the example exercise is to estimate the grade, here in per cent, of the central,

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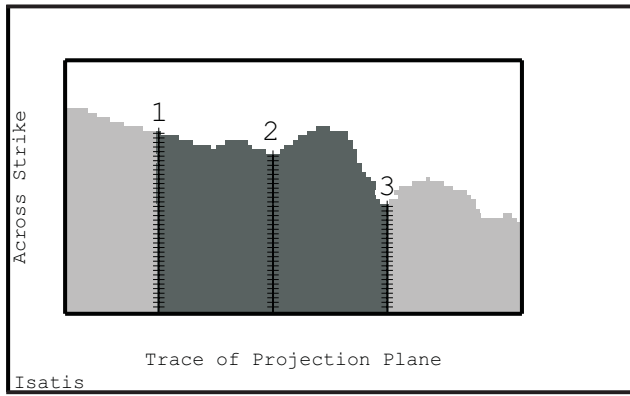


FIG 1 - Diagrammatic plan view illustrating 2D estimation process. NB: Please only refer to the above diagram in conjunction with illustrative example in text.

highlighted area using the information contained in samples 1, 2 and 3. Each of these samples has a length and a grade and, for simplicity, the samples are all oriented orthogonally to the longitudinal plane so that the length corresponds to a projected horizontal thickness of the mineralised interval.

The characteristics of the three samples are shown in Table 1. Note that grade appears to be correlated with the length of the interval. The grade of the central area is the ratio of the metal contained in that area to the tonnage (assuming that the bulk density is constant). Now, let:

- S be the surface area of the central area expressed in square metres
- Q be the metal contained in the central area
- T be the corresponding tonnage
- BD be the bulk density
- L denote thickness (which equates in the example to the length of the interval)
- A be the accumulation (thickness × grade)

The estimates of T and Q (respectively T* and Q*) are:

$$T^* = S \times BD \times \frac{(L_1 + L_2 + L_3)}{3} \text{ and } Q^* = S \times BD \times \frac{(A_1 + A_2 + A_3)}{3}$$

Numerically, $T^* = S \times BD \times 33$ and $Q^* = S \times BD \times 88.67$. The mean grade is the ratio Q^*/T^* and in our example:

$$m^* = \frac{88.67}{33} = 2.69\%$$

This mean is simply the average of the grades weighted by the length of the intervals. Note that the arithmetic mean (3D approach) of the grades would have given $m^* = 2.60$ per cent which is incorrect.

TABLE 1

Characteristics of 2D samples in illustrative example.

	Thickness (m)	Grade (%)	Accumulation (m %)
1	L ₁ = 40	g ₁ = 3	A ₁ = 120
2	L ₂ = 35	g ₂ = 2.8	A ₂ = 98
3	L ₃ = 24	g ₃ = 2	A ₃ = 48

Note: Thickness = Projected horizontal width of mineralisation

Now assume that the density is no longer constant but is correlated to the grade (see Table 2).

TABLE 2

Characteristics of 2D intervals with varying densities for the illustrative example.

	Thickness (m)	Grade (%)	Bulk density (t/m ³)	Accumulation (kg %/m ²)
1	L ₁ = 40	g ₁ = 3	BD ₁ = 4	A ₁ = 480
2	L ₂ = 35	g ₂ = 2.8	BD ₂ = 3.5	A ₂ = 343
3	L ₃ = 24	g ₃ = 2	BD ₃ = 2.5	A ₃ = 120

We now have:

$$T^* = S \times \frac{(BD_1 L_1 + BD_2 L_2 + BD_3 L_3)}{3} \text{ and}$$

$$Q^* = S \times \frac{(BD_1 A_1 + BD_2 A_2 + BD_3 A_3)}{3}$$

In our example, this gives $T^* = S \times 114.17$ and $Q^* = S \times 314.33$.

The resulting grade is thus $m^* = 2.75$ per cent, and this is the average of the grades weighted by length and density of each interval.

It is concluded that the absence of proper density weighting in this example induces a relative bias in metal of 2.2 per cent (globally). Such a bias may be even more pronounced depending on the anisotropy of the variogram models employed.

In conclusion, this simple illustrative example highlights the need to properly incorporate density in calculations when density is highly correlated to grade.

2D ESTIMATION – THE 1A SHOOT CASE STUDY

Regional geology

The 1A Shoot is a fault hosted, massive nickel sulfide deposit associated with the Perseverance disseminated deposit, which is about 12 km north of Leinster, in the North Eastern Goldfields of Western Australia. Nickel mineralisation in the area occurs on the eastern margin of the Agnew-Wiluna greenstone belt, a major nickel-sulfide province shown in Figure 2.

The stratigraphic sequence comprises tholeiitic basalt with minor gabbro, overlain by a thick sequence of dominantly felsic, volcanic and volcanoclastic rock with lesser mafic units, cherts, pelitic sediments and black shales (Gole, Barnes and Hill, 1988). Several komatiite sequences are intercalated within the felsic sequence, some of which contain large olivine accumulate units.

The greenstone belt is bound by granitoid rock with sheared and faulted contacts. The Perseverance area of the belt is truncated to the east by the Perseverance Fault (Figure 2), which forms part of the larger, craton scale Keith-Kilkenny lineament. The greenstone sequence has been disrupted by three major deformational events. The D1 phase involved north-south thrusting with associated isoclinal recumbent folding. During D2, ENE-WSW compression resulted in regional scale, north-northwest trending upright folds with intensely deformed zones. D3 deformation involved NNW sinistral and NNE dextral strike slip faulting (Libby *et al*, 1998).

Metamorphic grade decreases to the north, ranging from low amphibolite facies around the Perseverance area to prehnite-pumpellyite toward Wiluna. Peak metamorphic temperature in the Leinster area reached 550°C, with pressures up to 3 kb (Gole, Barnes and Hill, 1988). Hydrothermal alteration has been extensive throughout the belt, with the felsic

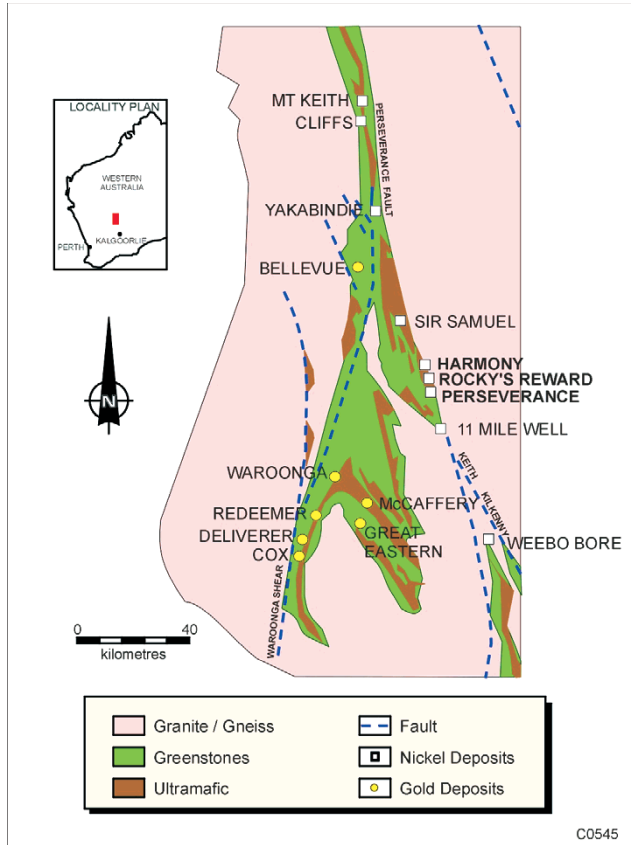


FIG 2 - Agnew-Wiluna greenstone belt.

lithologies altering to chlorite-biotite-actinolite assemblages, and the ultramafic sequences serpentinised with varying degrees of talc-carbonate alteration. Olivine is preserved in some areas, normally coincident with the larger adcumulate units.

Geology of the 1A Shoot

From a mining perspective, and for the purposes of this paper, '1A Shoot' refers only to the fault zone to the north of the main Perseverance disseminated nickel sulfide deposit. The footwall sequence at Perseverance consists of variably altered volcanic and volcanoclastic rock with minor mafic units. The Perseverance ultramafic complex consists of serpentinised mesocumulate flows which grade to the east into an adcumulate core. The 1A Shoot occurs within a north striking, steeply west dipping fault zone that occurs immediately to the west and to the north of the Perseverance ultramafic complex as shown in Figure 3. The fault zone is entirely within the felsic footwall sequence to the north of Perseverance, and extends through these units to at least the Rocky's Reward deposit, 2 km to the north. To the south, the fault zone forms the contact between the felsic sequence and the Perseverance ultramafic complex.

Nickel mineralisation is contained within remobilised, brecciated, massive sulfides and lithons of ultramafic rock with disseminated sulfides. The mineralisation is classified by project geologists as either 'massive' or 'ultramafic' on the basis of sulfide percentage and these terms are used in the remainder of this paper. Lithons of barren ultramafic and felsic rock also occur within the fault zone. The distribution of these units within the structure is highly complicated, with significant pinching and swelling of these units over very short distances, and many faulted offsets (Figure 4).

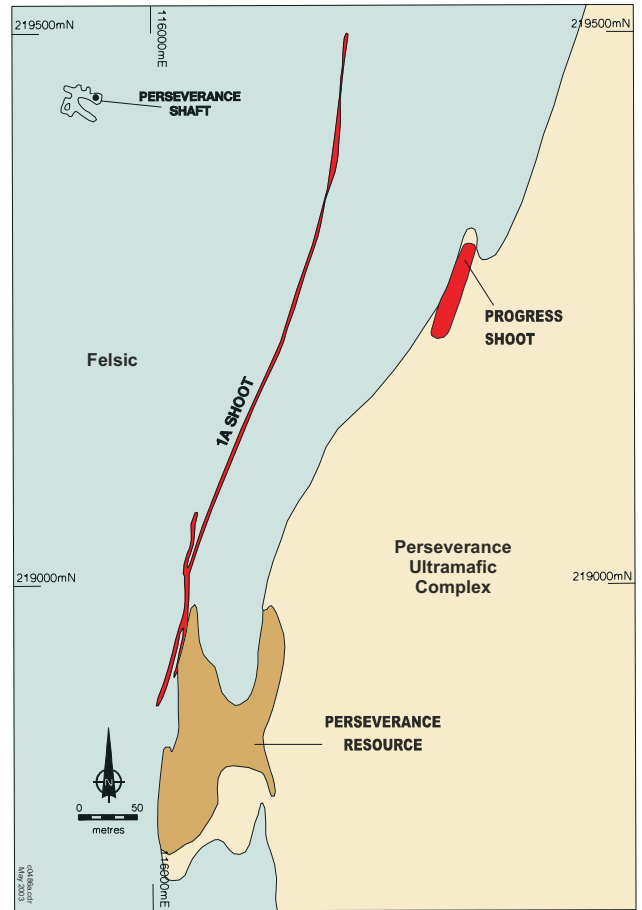


FIG 3 - Resource and adjacent geology (9700 mRL plan view).

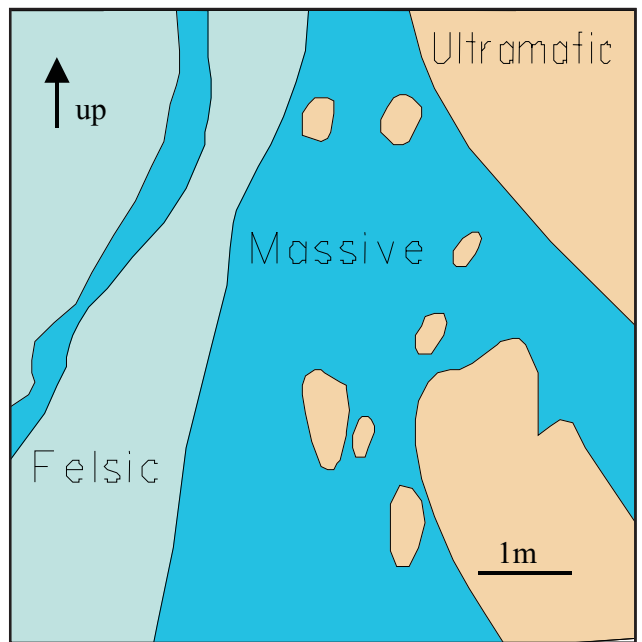


FIG 4 - 1A geological mapping – section.

Significant nickel mineralisation extends about 600 m vertically below 10 150 mRL (the natural surface is at approximately 10 520 mRL), with weaker mineralisation above the 10 150 mRL. Although the fault zone continues at depth, mineralisation appears to pinch out. The northern extent of the 1A Shoot mineralisation is poorly constrained and has not been definitively closed off by drilling. The 1A Shoot has been mined out above the 9900 mRL with active mining currently being conducted down to 9780 mRL.

Domaining

The key to the validity of any resource model is appropriate definition of domains by project geologists. The variables to be estimated within these domains should display a level of statistical homogeneity compatible with an assumption of stationarity (see Journel and Huijbregts, 1978: Chapter II, for discussion of stationarity). For the 1A case study, the following were deemed to be important when defining domains:

1. geological interpretation is based primarily on diamond drilling (core) data;
2. underground mining has allowed face and back mapping to be extensively integrated into these geological interpretations;
3. the 1A is mostly a single continuous 'layer' of massive sulfide/ultramafic within a subvertical shear structure; and
4. this layer is locally thinned, or conversely, thickened (ie it pinches and swells) and these changes of geometry can be very rapid in both plan and sectional view (see Figure 4).

One approach to 2D estimation at 1A would be to use a single domain: accumulating all massive and ultramafic mineralisation. This domaining decision was tested from a stationarity view point and it was concluded that the result would be to mix two distinctive, though adjacent, domains. It was therefore rejected.

1A Shoot data

The main statistics for the 1A Shoot dataset are presented in Tables 3 and 4. Note that in Tables 3 and 4 'accumulation' refers to 'triple accumulation' ie the product: grade by thickness by bulk density.

Table 4 shows:

1. the very high level of correlation between the tonnage and metal variables for each geological unit (highlighted as bold numbers): this correlation approaches 'intrinsic correlation' (see Rivoirard, 1994 p 10, 2001) between the two variables and is discussed more fully below; and
2. the lack of correlation between metal (and/or tonnage) in massive and ultramafic (highlighted as italic numbers): this is critical because it establishes the apparent absence of link between the distributions of metal in the two mineralised units – providing an argument for their independent estimation.

The statistics for nickel (Ni), density (BD) and interval length for the 1A samples and the scatter diagram in Figure 5 clearly show the high level of correlation between nickel grade and density. As shown in the preceding illustrative example, such pronounced correlation makes it essential that bulk density is incorporated in the weighting of the estimates for the case study. Figure 5 also highlights how Ni and BD can be used to broadly separate the groups of samples that correspond to the three geological units.

Variography

Variography was performed on the 2D variables thickness and accumulation. Accumulation and thickness were interpreted to be intrinsically correlated, ie perfectly correlated. This is important, because in such a case a proportional variogram model is applicable (ie the variogram for accumulation, the variogram for

TABLE 3

Statistics for 2D nickel variables in 1A.

Variable	Count	Minimum	Maximum	Mean	Variance	CV
Accumulation felsic	109	0.00	1.52	0.06	0.06	3.81
Accumulation massive	109	0.00	168.67	30.74	1342.07	1.19
Accumulation ultramafic	109	0.00	126.14	12.63	310.42	1.40
Thickness felsic	109	0.00	8.21	0.44	1.73	3.01
Thickness massive	109	0.00	26.20	5.09	32.14	1.11
Thickness ultramafic	109	0.00	66.97	8.22	122.58	1.35

TABLE 4

Correlation matrix for 2D nickel variables in 1A.

Variable	Accumulation felsic	Accumulation massive	Accumulation ultramafic	Thickness felsic	Thickness massive	Thickness ultramafic
Accumulation felsic	1.00	0.14	- 0.08	0.70[†]	0.14	- 0.08
Accumulation massive	0.14	1.00	- 0.02	0.18	0.99	0.01
Accumulation ultramafic	- 0.08	-0.02	1.00	- 0.09	- 0.04	0.97
Thickness felsic	0.70	0.18	- 0.09	1.00	0.18	- 0.09
Thickness massive	0.14	0.99	- 0.04	0.18	1.00	-0.01
Thickness ultramafic	- 0.08	0.01	0.97	- 0.09	- 0.01	1.00

[†] 'jumps' to 0.85 when one pair (0.05, 8.21) is removed.

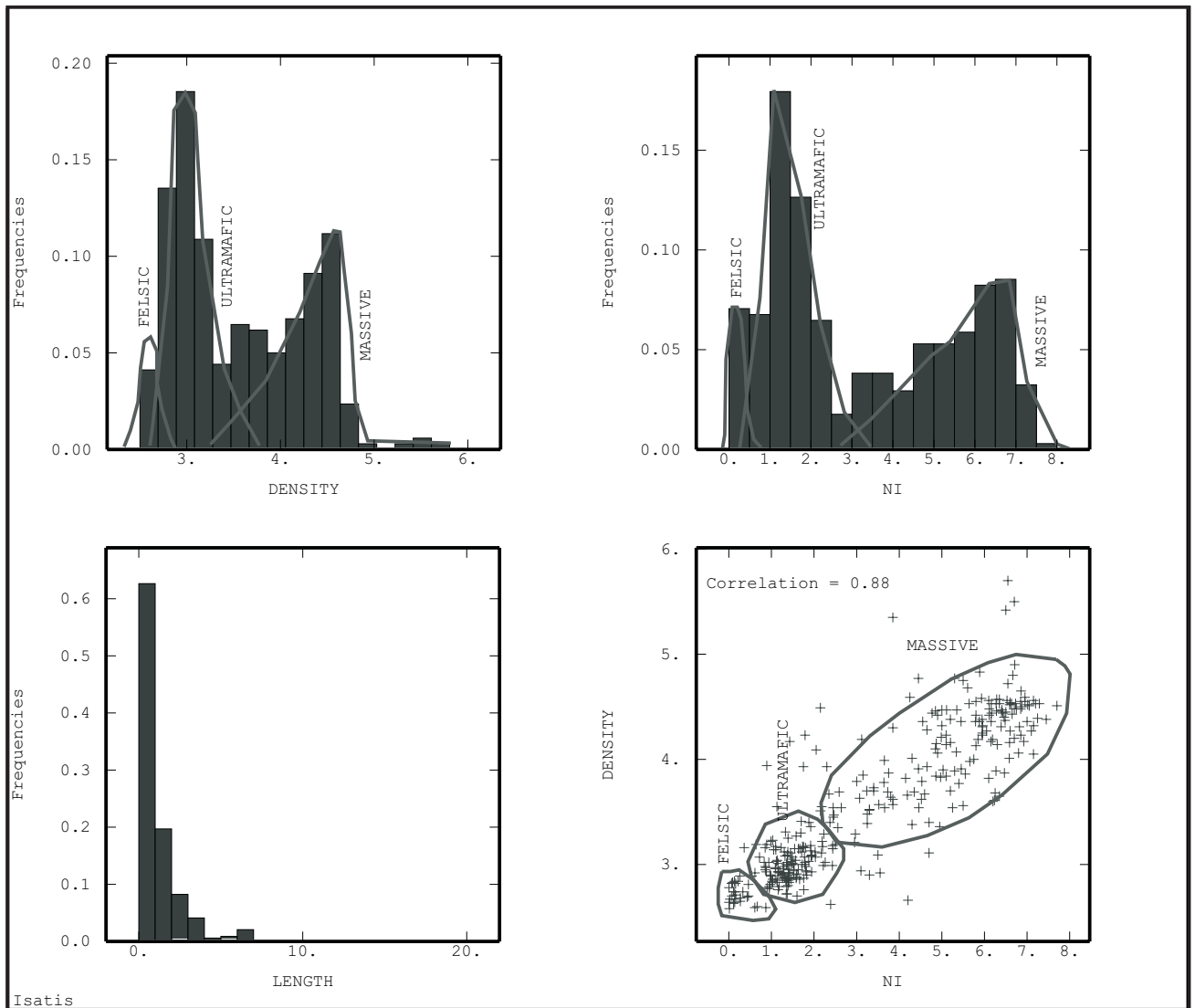


FIG 5 - Histogram for nickel, horizontal width and bulk density for 1A intervals.

thickness and their cross-variograms are all proportional to the same model – see Rivoirard 2001). The advantage of using a proportional model, when (as in the case of 1A – see Figure 6) it can be demonstrated that it is acceptable, is that it ensures consistency of the calculated mean grades in different portions of the lode. In that case independent kriging of accumulation and thickness – which was adopted for 1A – is equivalent to their co-kriging.

The mean grade of a portion of the lode is estimated as the ratio: estimated metal/estimated tonnage (or Q^*/T^*). If the variogram models are proportional for thickness and accumulation then within the co-kriging search neighbourhood two *identical* sets of weights (λ_α) will result and we have:

$$m^* = \frac{Q^*}{T^*} = \frac{S \times \sum \lambda_\alpha Ni_\alpha BD_\alpha HT_\alpha}{S \times \sum \lambda_\beta BD_\beta HT_\beta} = \sum \frac{\lambda_\alpha BD_\alpha HT_\alpha}{\sum \lambda_\beta BD_\beta HT_\beta} Ni_\alpha$$

Where $\frac{\lambda_\alpha BD_\alpha HT_\alpha}{\sum \lambda_\beta BD_\beta HT_\beta}$ sum up to 1

Where HT denotes the projected horizontal thickness. Note that m^* is actually a weighted average and will thus produce consistent results from a practical view point.

Resource estimation

The data spacing in the well informed parts of the 1A deposit is nominally 20×20 m. 2D estimation was performed on 10×10 m blocks. This block size was the result of a compromise between estimation quality and mine design considerations and was tested quantitatively through quantitative kriging neighbourhood analysis.

At the time of the 1A study there was no wireframe of the mineralised lode. The very high short-scale variability of geometry (thickness has a significant relative nugget effect, about 40 per cent) would make such a wireframe of questionable validity in any case. As noted previously, and shown in Figure 4, the local geometry is poorly constrained away from mined areas and the drilling is insufficient to allow detailed understanding of the local distribution of mineralisation. Consequently, significant uncertainty is attached to the estimation of mineralised volumes and subsequently of tonnage and metal. In particular, the question of how far values can be extrapolated is clearly an important one. A decision was made to create 2D grids that extended 50 m beyond the last known data point and to fill in these grids entirely. The estimates would subsequently be constrained by geological perimeters to limit the extent of the mineralised lodes based on a first order geological interpretation.

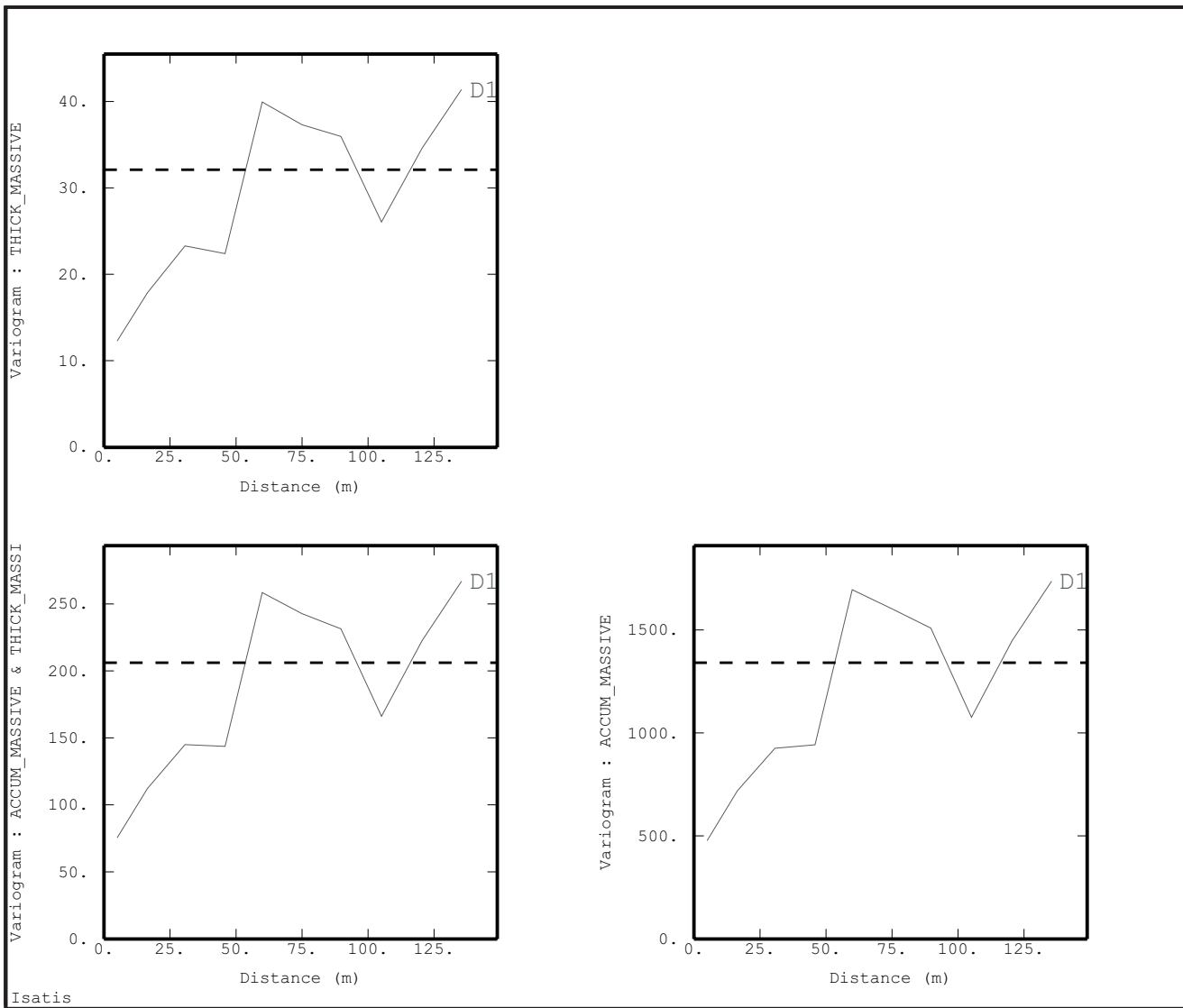


FIG 6 - Omni-2D variogram for accumulation and thickness in massive 1A.

TABLE 5
Neighbourhood parameters for massive1A.

	1st run	2nd run	3rd run	4th run
Radius easting (m)	40	80	120	300
Radius RL (m)	40	80	120	300
Minimum samples	10	8	5	5
Angular search (Q: quadrant; O: octant)	Q	Q	Q	Q
Optimum per angular sector	5	5	5	5
Discretiation (x, y)	(10 × 10)	(10 × 10)	(10 × 10)	(10 × 10)

The use of staged kriging runs with progressively larger neighbourhoods was considered necessary to fill in the grids.

The kriging neighbourhoods were optimised according to the strategy detailed in Vann, Jackson and Bertoli (2003) and are presented in Table 5.

Typical kriging test results for blocks estimated using the first two neighbourhoods are detailed in Table 6.

In general, for the purpose of mine design, it is also necessary to relocate 2D blocks back into 3D space. In the 1A case study,

an estimate of the easting of each block was performed by kriging of the offset distance between the mid-point of the shoot intercept and the nominal easting plane (116 000 mE). This kriging was run using a range of variogram models (ordinary kriging with cubic model, spline model kriging – see Chiles and Delfiner, 1999 pp 272-276), which all impose a high degree of spatial continuity on the offset distance compatible with the 1A Shoot planar structure. The final estimate of easting was controlled and validated by the project geologists.

TABLE 6

Typical kriging test result for blocks estimated at first and second runs in 1A.

	Accumulation	Thickness
Slope of regression	1.01	1.01
Correlation coefficient	0.90	0.90
Weight of the mean	0.5%	3%
Cumulative sum of negative weights	- 5.00%	-2.00%

CONCLUSIONS

The same key component steps are necessary in undertaking a 2D estimate as for sound 3D estimation: definition of domains, validating the stationarity decision, experimental variography and model fitting and quantitative neighbourhood analysis. To reduce the risk inherent in any estimation exercise, the same attention must be paid when following a 2D approach to the definition of the variogram model and the inference of its parameters as for a 3D exercise. In particular, variography and subsequent optimisation of neighbourhood search parameters are vitally important and potentially sensitive steps.

The particularities of 2D estimation revolve around the variables that are used to perform resource modelling. The variable usually estimated in mining, ie grade, loses its precedence in favour of the additive variables that capture the essence of the 2D estimation problem:

1. tonnage – in our case study this is controlled by the thickness of the mineralised body, or more generally to an appropriate projection thickness; and
2. metal – also referred to as accumulation, the product of grade by thickness, potentially weighted by bulk density if the latter is correlated to the grade and length of the mineralised intercept.

In the interests of avoiding biases due to violation of additivity considerations, the appropriateness of a 2D approach is beyond debate. The main issue, in fact, lies in the *practical definition of the thickness and accumulation variables*. This definition requires a careful consideration of not only the grade and length of the mineralised intercepts but also their orientation, if a meaningful, appropriate projection is to be decided upon.

Once these variables are defined, the tools of linear geostatistics can be deployed. In particular the variographic

analysis often exhibits a certain level of correlation between the metal and tonnage variables. This correlation needs to be properly modelled to allow co-kriging to be performed leading to further enhanced consistency of the resulting resource estimate.

In summary, the authors strongly recommend that narrow seam, layer or vein like geometry be estimated in 2D wherever this is practical, because the consequence of 3D estimations in these cases (in particular global and local biases, but also geometric issues) are usually unacceptable.

As an aside, it is worth stating that an additional practical problem with 3D approaches when applied to narrow seams, layers or veins arises from the difficulties of dealing with geometry.

These problems are overcome when a 2D approach is adopted. Although the primary motivation for 2D approaches is the issue of additivity, these secondary advantages are significant in many if not most cases.

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REFERENCES

- Chiles, J P and Delfiner, P, 1999. *Geostatistics, Modelling Spatial Uncertainty* (Wiley Series in Probability and Statistics: Wiley Inter-Science: New York).
- Gole, M J, Barnes, S J and Hill, R E T, 1988. The Geology of the Agnew Nickel Deposit, Western Australia, *CIM Bulletin*, 82:46-56.
- Journel, A G and Huijbregts, Ch J, 1978. *Mining Geostatistics* (Academic Press: London).
- Libby, J W, Stockman, P R, Cervoj, K M, Muir, M R K, Whittle, M and Langworthy, P J, 1998. Perseverance Nickel Deposit, in *Geology of Australian and Papua New Guinean Mineral Deposits* (Eds: D A Berkman and D H Mackenzie) pp 321-328 (The Australasian Institute of Mining and Metallurgy: Melbourne).
- Rivoirard, J, 1994. *Introduction to Disjunctive Kriging and Non-linear Geostatistics* (Clarendon Press :Oxford).
- Rivoirard, J, 2001. Which Models for Collocated Cokriging? *Journal of Mathematical Geology*, 33:117-131.
- Vann, J, Jackson, S and Bertoli, O, 2003. Quantitative Kriging Neighbourhood Analysis for the Mining Geologist — A Description of the Method with Worked Case Examples, in *Proceedings Fifth International Mine Geology Conference*, pp (The Australasian Institute of Mining and Metallurgy: Melbourne).

